

To: betaveros@beta.vero.site

Subject: hunthunthunt

Congratulations, you have just received a mini-hunt written specifically for you! (You as in betaveros. If you are someone else, pretend you're betaveros.)

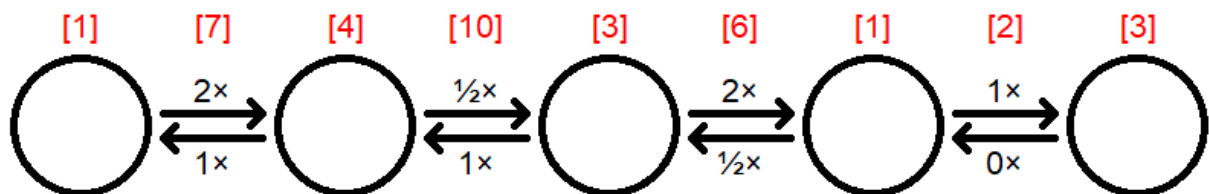
Hopefully you received this file inside a folder containing several other files. If you don't have the other files, check with whoever that sent you this file. The only important thing in the folder besides this file is the file `brackets.py`, but the folder also has several other files that will make your life easier.

Also, yes, this is an entire mini-hunt. Five puzzles plus a meta; the meta is just below. It's likely that you have already figured out who created this monstrosity from just reading this text, but not that I'm trying to hide myself anyway. Good luck!

Obviously everything above and including this line are not part of any puzzle.

## Not A Signature Metapuzzle

*All of these mystical creatures are so unique, they cannot be put into categories. However, there is something even more majestic.*



### Creatures

Lv.35: Allied Occupation

Lv.37: Remix the Past

Lv.40: Rough Sketches

Lv.48: Brackets

Lv.53: Sovereignty

# Allied Occupation

*I lost the exact rules for these puzzles; I hope you can figure them out.*

**Examples, with the unique solution and an example incorrect solution**

2		2		
3				1
		3		3

2	6	6	6	6
2	6	2	2	6
3	4	3	3	1
3	4	3	1	3
3	4	4	3	3

2	1	4	4	2
2	3	2	4	2
3	3	2	4	1
4	4	3	3	3
1	4	4	2	2

			1	
	2			
	1	2	2	
			1	
3				

4	4	4	1	5
2	2	4	5	5
5	1	2	2	5
5	5	5	1	5
5	3	3	3	1

6	2	7	1	7
6	2	7	7	7
6	1	2	2	7
6	6	6	1	7
1	3	3	3	1

	1			3
				1
3				
5			4	

7	1	3	3	3
7	7	7	7	1
3	3	5	7	4
3	5	5	7	4
5	5	1	4	4

2	1	6	3	3
2	6	6	3	1
3	6	6	6	4
3	3	5	5	4
5	5	5	4	4

	5			
			4	
	5			
			6	

10	5	4	4	4
10	5	5	4	6
10	5	10	6	6
10	5	10	10	6
10	10	10	6	6

5	5	5	5	4
4	5	1	4	4
4	4	4	6	4
5	5	6	6	6
5	5	5	6	6

4		6		
	4		5	
		4		4

4	6	6	6	6
4	3	3	5	6
4	4	3	5	6
3	5	5	5	4
3	3	4	4	4

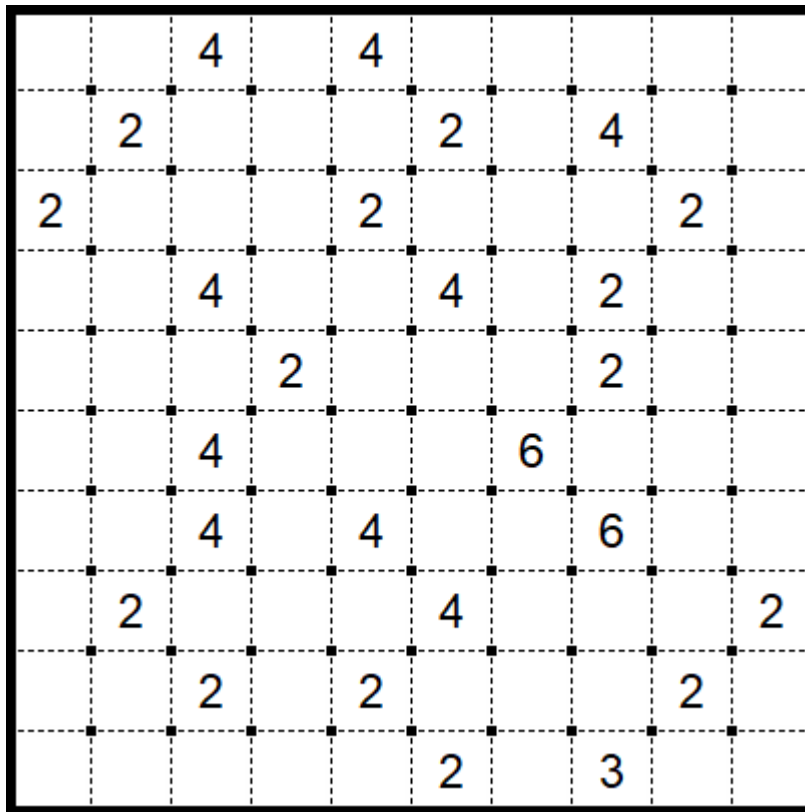
4	6	6	6	6
4	6	6	5	5
4	4	5	5	2
3	3	5	4	2
3	1	4	4	4

		V		
	R			N
V				X
K			Q	
		R		

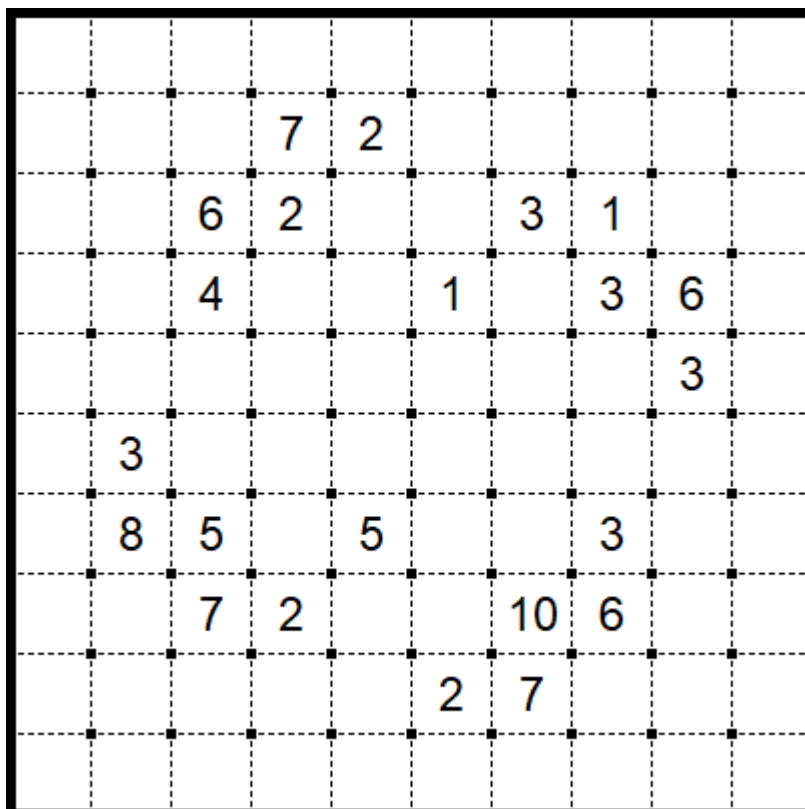
		S		
	O			L
U				T
I			O	
		N		

SOLUTION

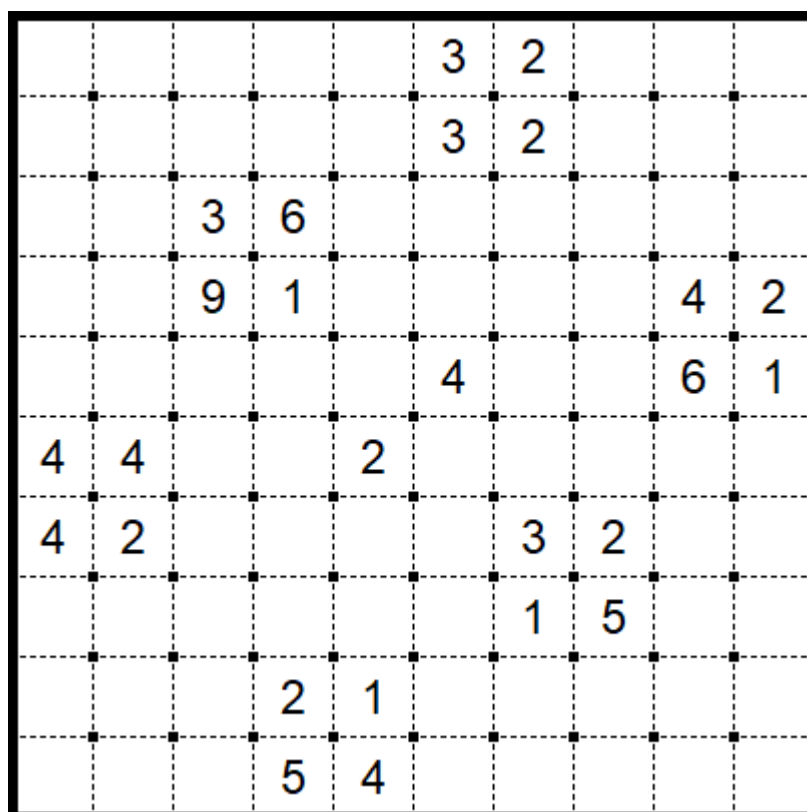
**Puzzles** (each puzzle is flood-fill friendly and has a PUZ-PRE link)



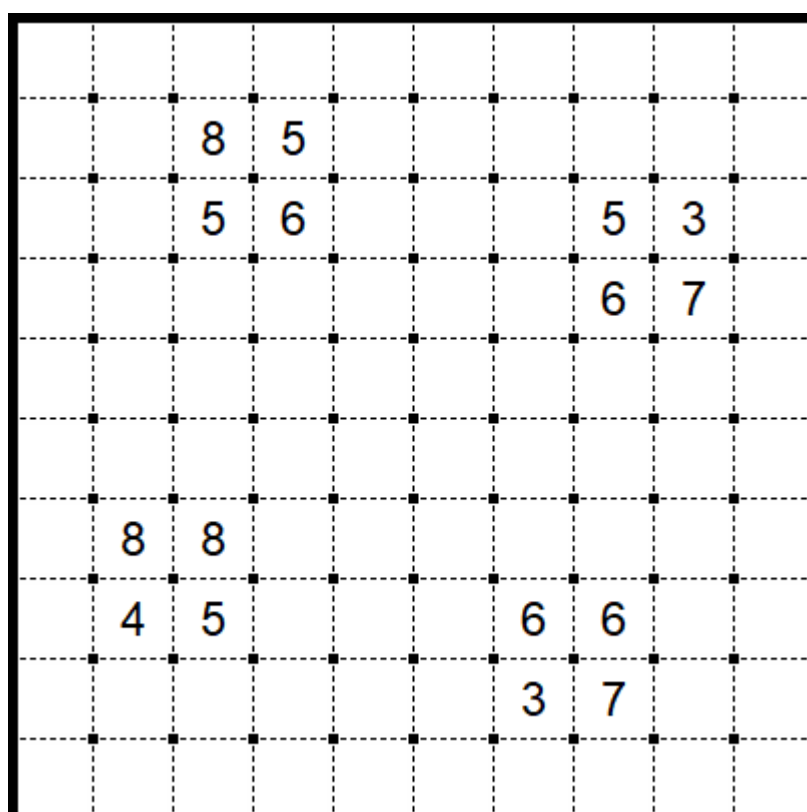
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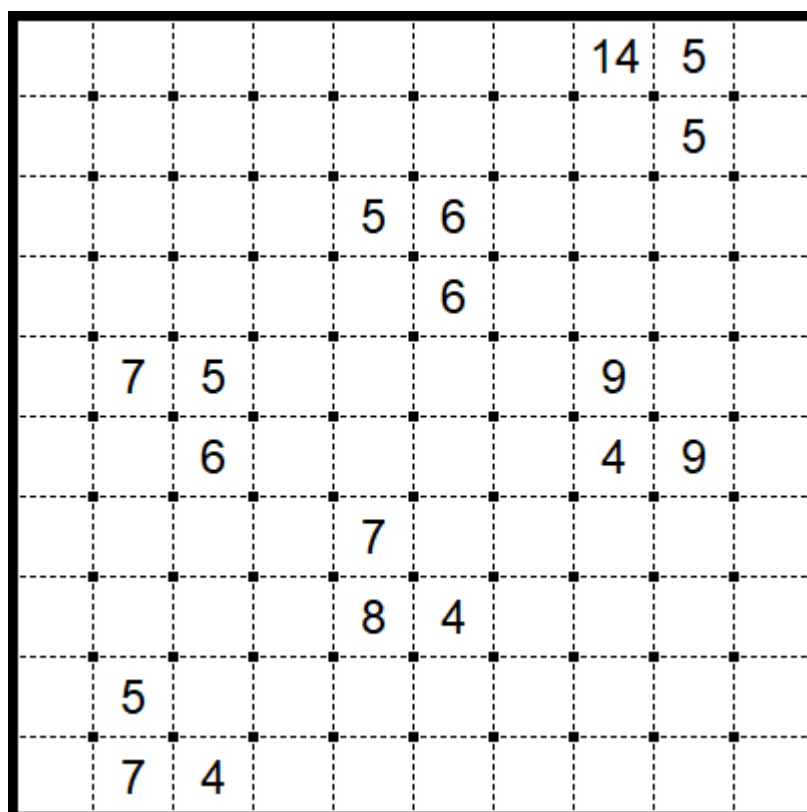
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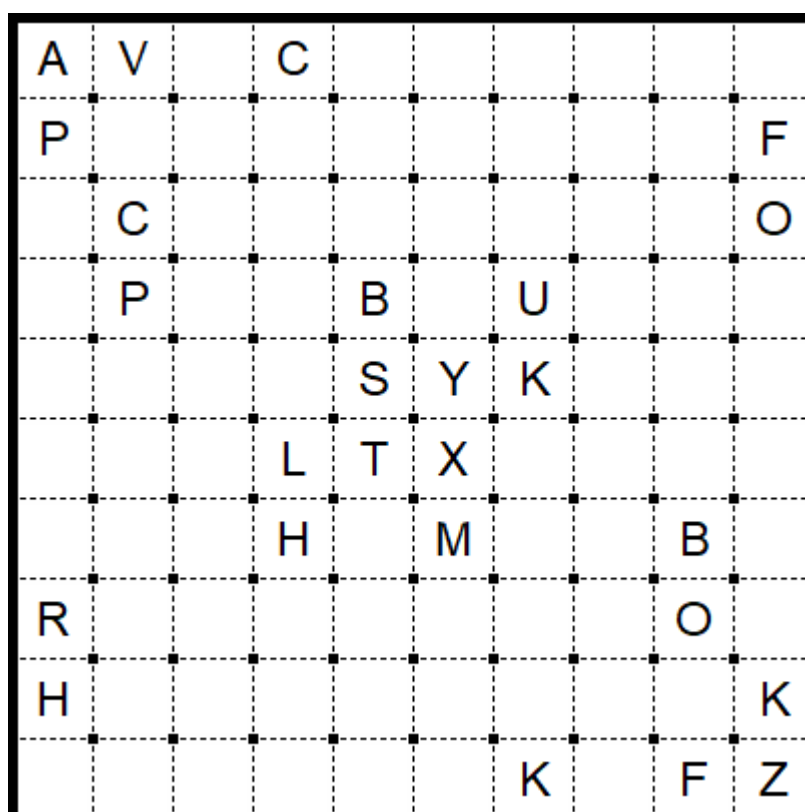
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# Remix the Past

*These puzzles seem quite familiar...*

## Twists

- It is not the start of the sequence.
- One part of the statements should be negated.
- Use directional buttons.
- The constant is different.
- The encoding is keyboard-based.
- The questions are jumbled.
- The starting permutation is different.
- There are two valid answers.

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...the American low-cost carrier (without "Airlines") known for its humor, and the only start node of this puzzle.	...the title of a numbered chapter in Cytus where all Hard charts in it are rated Level 7 or above (Cytus Alive songs excluded).	...the kingdom in Super Mario Odyssey that has five story missions.
...the Remodel variant in Dominion: Intrigue.	...the five-letter character class present in the Player's Handbook of Dungeon & Dragons 5th edition.	...the Nikoli puzzle type where one number in each region is wrong.
...the function of the button in a PuzzleScript game that cannot be overridden.	...the doubly color-bound piece in monochromatic chess.	...the country where MIT Mystery Hunt has taken place.

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March 3, March 4, March 5, April 1, April 4, April 5, May 1, May 3, June 2, June 4

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Game observer (3|5)  
Neither prime nor unit (0|8)  
Book of definitions (2|7)  
Bishop and knight compound (2|7)  
Magic: The Gathering card type (2|8)  
Cube's dual (1|8)  
In a single shade of color (4|5)  
Vlaada's team-based board game (0|8)  
Weed killer (5|3)

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RDMAON  
RONAMD  
RONADM  
RODANM  
RONMAD  
RDOMNA  
RDMONA  
RONMDA

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August 2, August 3, August 4, August 5, September 1, September 4, September 7,  
September 8, October 4, October 6, November 2, November 6, November 8,  
December 5, December 6, December 7, December 8

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- ...the board game in the Oniverse series up to 2018 that includes dice.
  - ...the Chinese winner of The International that was hosted in KeyArena.
  - ...the company that owns the main character in the official game of 2014 Winter Olympics.
  - ...the word meaning "erect" that is the concatenation of two directions that also indicates the location of this node.
  - ...the Greek letter that is a proper suffix of another Greek letter.
  - ...the name of the difficulty easier than Hard in Touhou Project: Hidden Star of Four Seasons.
  - ...the quest location in 2017 MIT Mystery Hunt (without "The") where you needed to find four winning positions.
  - ...the title of a Generation III main series Pokémon game (without "Pokémon") where Sableye can be caught in the wild.
  - ...the way out from this middle node.
-



AMNROD  
AMNRDO  
AMDNOR  
AMNRDO  
ANMODR  
AMRNOD  
ANDMOR  
ANRMDO  
ANRMOD  
ANMODR  
AORDMN  
ANRDOM  
AMRNOD  
ANDMOR  
AMNRDO  
AMNRDO  
AMRNOD  
ANMORD  
AMDNOR  
ANRMOD

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.  
. .  
.

Climate of Indonesia (3|3)

Not fact (1|4)

e.g. kangaroos and wombats (5|2)

Italian town square or Q&A web service (0|4)

Toll road (4|2)

(Very stupid) (7)

(Dog-like) (6)

(Boss song in a main chapter of Lanota) (7)

(Having qualities associated to a female) (8)

(Floor halfway between two floors) (9)

(Gloomy) (9)

Mathematical principle, also known as Dirichlet's box (0|8)

Protection (7|2)

Tagging out a runner before the pitch (0|5)

Mascot of Pokémon (0|5)

Common name of the order Mecoptera (4|5)

Pilot's space in aircraft (4|1)

Sudden insight (1|5)

.  
. .  
.

# Rough Sketches

*Sorry, I ran out of time, so I could only give rough sketches of the solutions.*

## Problem 11.7

Drop the perpendicular from  $C$  to  $AB$  and let the base be  $D$ . We have  $\angle ADC = \angle ACB$  and  $\angle DAC = \angle CAB$ , so  $\triangle ADC \sim \triangle ACB$ . So we get  $\frac{AD}{AC} = \frac{AC}{AB}$ , or  $AD = b^2/c$ . Similarly we get  $BD = a^2/c$ . But  $AD + BD = AB$ .

## Problem 11.7.2.10

For existence, if  $s$  is 1 or prime then we're done. Otherwise  $s = ab$  for some  $a, b < s$ , so induct to get  $a = p_1 \dots p_m$  and  $b = q_1 \dots q_n$ , so  $s = p_1 \dots p_m q_1 \dots q_n$ . For uniqueness, if  $s = p_1 \dots p_m = q_1 \dots q_n$ , by Euclid's lemma  $p_1$  divides some  $q_i$ ; without loss of generality it's  $q_1$ . Then  $p_1 = q_1$ , so induct on  $s/p_1$ .

## Problem 11.5

Consider pairs of  $(e, v)$  where  $e$  is an edge and  $v$  is an endpoint of  $e$ . Each edge is counted twice, so the number of pairs is even. However, each  $v$  is counted  $\deg v$  times. So the sum of all degrees is even; if there were an odd number of odd vertices the sum would be odd.

## Problem 6.7.10

Consider the quadratic  $\sum (u_i x + v_i)^2$ . It has at most one real root, so its determinant  $4(\sum u_i v_i)^2 - 4(\sum u_i^2)(\sum v_i^2)$  is non-positive.

## Problem 12.5.7

Without loss of generality suppose  $f(a) < u < f(b)$ . Define  $S$  as all  $x \in [a, b]$  where  $f(x) \leq u$ .  $S$  is nonempty because  $a \in S$  but is bounded above by  $b$ , so  $c = \sup S$  exists. Let  $\epsilon > 0$ ; there exists  $\delta > 0$  such that whenever  $|x - c| < \delta$ ,  $|f(x) - f(c)| < \epsilon$ . Since  $c$  is supremum of  $S$ , there exists  $a^* \in (c - \delta, c] \cap S$ , so  $f(c) < f(a^*) + \epsilon \leq u + \epsilon$ . However, for all  $b^* \in (c, c + \delta)$ ,  $b^* \notin S$ , so  $f(c) > f(b^*) - \epsilon > u - \epsilon$ . Now take  $\epsilon \rightarrow 0$ .

## Problem 5.8.7

Let  $(a_i, b_i)$  be the lengths of longest increasing/decreasing subsequence ending on  $x_i$  respectively. Then no pair is repeated. There aren't enough pairs in  $[1, r - 1] \times [1, s - 1]$ , so there is some pair with  $a_i \geq r$  or  $b_i \geq s$ .

### **Problem 3.8.7**

For each unit vector  $p$ , define  $\pi(p)$  as the half-space that is normal to  $p$ , in direction of  $p$ , and contains exactly half of  $A_n$ ; if there are multiple such spaces, take the middle one. Define  $f(p) = (\text{vol}(A_1 \cap \pi(p)), \dots, \text{vol}(A_{n-1} \cap \pi(p)))$ . Then  $f$  is continuous; by Borsuk-Ulam theorem, there exists  $p$  where  $f(-p) = f(p)$ .

### **Problem 13.10**

It suffices to prove the upper bound. Let  $\sigma$  be such that  $\sum_i x_{\sigma(i)} y_i$  is maximum; if there are many such  $\sigma$ , take one with the most fixed points. Suppose  $\sigma$  is not the identity, then there exists a smallest non-fixed point  $j$  of  $\sigma$ , and a  $k > j$  where  $\sigma(k) = j$ . Then the permutation  $\sigma'$  obtained by swapping  $j$  and  $k$  will either give a larger sum or fix more elements.

### **Problem 2.6.5.7**

Let  $X$  be the space of all labelings of  $G$  with  $k$  colors, and give it the product topology  $k^{V(G)}$ . By Tychonoff's theorem  $X$  is compact. For each finite subgraph  $H$ , let  $X_H$  be the subset of  $X$  that colors  $H$  properly. Then  $X_H$  is closed and the family of all  $X_H$ 's have finite intersection property; by compactness they have nonempty intersection. Let  $c$  be a member in the intersection. Since  $c \in X_e$  for each edge  $e$  of  $G$ ,  $c$  colors  $G$  properly.

### **Problem 9.5**

For each  $x$ , define  $\alpha(x) = \min_{S \in \mathcal{F}, x \notin S} w(S) - \min_{S \in \mathcal{F}, x \in S} w(S \setminus \{x\})$ . Then  $\alpha(x)$  doesn't depend on  $w(x)$ . As  $w(x)$  is taken uniformly from  $\{1, 2, \dots, N\}$ , we get  $\alpha(x) = w(x)$  with probability  $\leq 1/N$ . So the probability  $\alpha(x) = w(x)$  for some  $x$  is  $\leq n/N$ . Now, if there are two sets  $A, B \in \mathcal{F}$  with the same minimum weight, then for any  $x \in A \setminus B$ , we have the equality  $\alpha(x) = w(B) - (w(A) - w(x)) = w(x)$  that only happens with probability  $\leq n/N$ .

### **Problem 5.1.7**

The system  $\sum_{i=1}^{d+2} a_i x_i = 0$  and  $\sum_{i=1}^{d+2} a_i = 0$  has  $d + 1$  equations (one for each dimension  $d$ , plus the last equation) but has  $d + 2$  unknowns, so there exists a solution  $a_i$ 's not all zero. Let  $X, Y$  be the set of indices where  $a_i \geq 0$  and  $< 0$  respectively. Then we have  $\sum_{i \in X} \frac{a_i}{A} x_i = \sum_{i \in Y} \frac{-a_i}{A} x_i$  where  $A = \sum_{i \in I} a_i$ , and each side is a convex combination of points  $\{x_i\}_{i \in X}$  and  $\{x_i\}_{i \in Y}$ , so this is a point in the convex hulls of  $\{x_i\}_{i \in X}$  and  $\{x_i\}_{i \in Y}$ .

### **Bonus problem**

By Definitions 6.5, 1.1, 5.4, 8.2, 2.4, 10.3, 2.1, 7.4, 1.7, 4.3, 3.3, 6.3, 5.1, 4.7, 9.3, 10.4, 8.3, 11.5, 3.8, 11.4, 7.6, 9.2, we get the answer.

# Brackets

*How did the value blow up with such a short expression?*

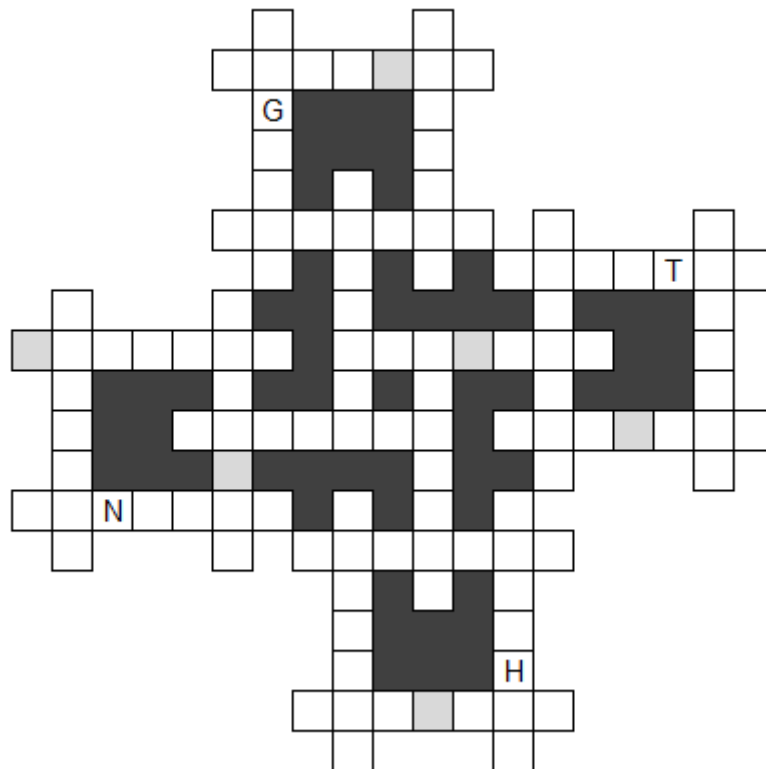
Run the Python 3 file `brackets.py`. You need Python 3.5.1+.

Please treat the file as a black box. For your enjoyment, do not inspect the source code of the file. (In a more serious hunt, this file would be hosted server-side and solvers would only be able to interact through a webpage.)

These instructions are not part of the puzzle.

# Sovereignty

*Despite the curses inflicted by our neighboring kingdom, our markets are flourishing and our laboratories are producing exciting results! Now if only we can figure out what these results mean.*



- Consultant that gives you the greens
- Strengthen bridges into tunnels and trading posts
- Two smaller warehouses as underground prison
- Pretty lawns make you thick
- Bargainer that doesn't bring you golds and hoards at the same time
- Cheaper prices through road
- Collection of books to expand your options
- Watch unknowns thrown away
- Feed that comes with shepherds
- Lowly person can become another entry
- Egg boiler weakening over time
- The first of its kind to change
- Can throne a staff
- Professor Oak's student
- Manager and trasher
- Coach from another entry
- Hunter bringing your package early
- Can turn into a bat, so always turns into a bat
- Rural town shouldn't be an idiot
- Rival isn't satisfied with coppers